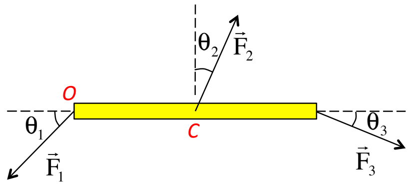
Problem 11.5

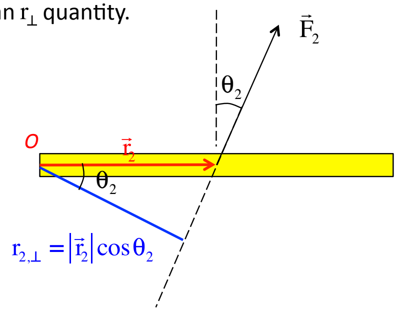
A system of forces is shown in the sketch.

a.) Determine the net torque about *Point O*.

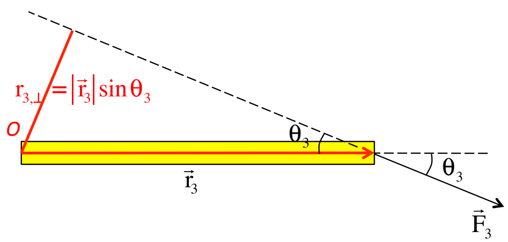


Because $\vec{F}_1$ acts through *Point O*, it's torque will be zero.

Remember: The torque due to $\vec{F}_2$ requires an $r_\perp$ quantity. The vector $\vec{r}$ is drawn from the *point about which the torque is being taken* (*Point O* in this case) to where the force acts. The component $r_\perp$ is the shortest distance between *Point O* and the *line of the force*. And because the torque motivates the beam to rotate *counterclockwise* about *O*, it will be a *positive* torque.



The torque due to $\vec{F}_3$ requires an $r_\perp$ quantity, which is shown in the sketch to the right, and because it motivates the beam to rotate *clockwise* about *O*, it will be a *negative* torque.

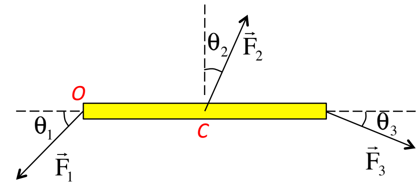


Keeping track of the signs, the net torque about *Point O* is:

$$\begin{aligned}
\vec{\Gamma}_{net,O} &= \vec{\Gamma}_1 + \vec{\Gamma}_2 + \left(-\vec{\Gamma}_3\right)\\
&= 0 + r_{2,\perp}\left|\vec{F}_2\right| - r_{3,\perp}\left|\vec{F}_3\right|\\
&= 0 + \left(\left|\vec{r}_2\right|\cos\theta_2\right)\left|\vec{F}_2\right| - \left(\left|\vec{r}_3\right|\sin\theta_3\right)\left|\vec{F}_3\right|\\
&= \left[(2.00\text{ m})\cos 30^\circ\right](25.0\text{ N}) - \left[(4.00\text{ m})\sin 20^\circ\right](10.0\text{ N})\\
&= 30.0\text{ N}\bullet\text{m}
\end{aligned}$$

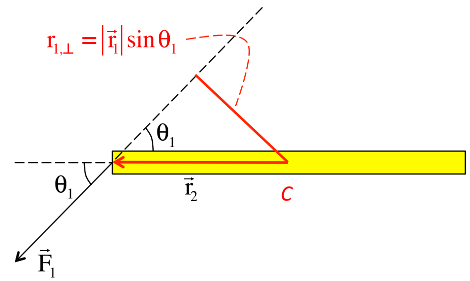
Because the value is *positive,* the net torque will be counterclockwise.

b.) Determine the net torque about *Point C*.

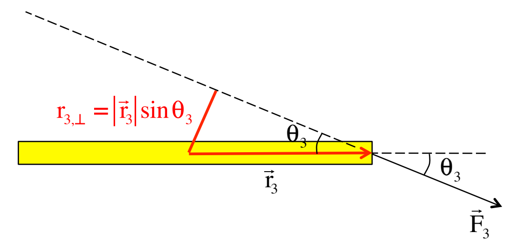


Because $\vec{F}_2$ acts through *Point C*, it's torque will be zero.

The torque due to $\vec{F}_1$ requires an $r_\perp$ quantity, which is shown in the sketch to the right, and because it motivates the beam to rotate *counterclockwise* about *C*, it will be a *positive* torque.



The torque due to $\vec{F}_3$ requires an $r_\perp$ quantity, which is shown in the sketch to the right, and because it motivates the beam to rotate clockwise about *C*, its torque will be *negative*.

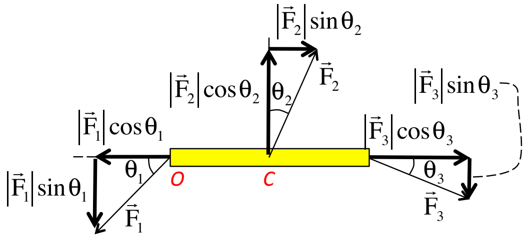


Keeping track of the signs, the net torque about *Point C* is:

$$\begin{aligned}
\vec{\Gamma}_{net,C} &= \vec{\Gamma}_1 + \vec{\Gamma}_2 + \left(-\vec{\Gamma}_3\right)\\
&= r_{1,\perp}\left|\vec{F}_1\right| + 0 - r_{3,\perp}\left|\vec{F}_3\right|\\
&= \left(\left|\vec{r}_1\right|\sin\theta_1\right)\left|\vec{F}_1\right| - 0 + \left(\left|\vec{r}_3\right|\sin\theta_3\right)\left|\vec{F}_3\right|\\
&= \left[(2.00\text{ m})\sin 45^\circ\right](30.0\text{ N}) - \left[(2.00\text{ m})\sin 20^\circ\right](10.0\text{ N})\\
&= 35.6\text{ N}\bullet\text{m}
\end{aligned}$$

Because the value is *positive,* the net torque will be counterclockwise.

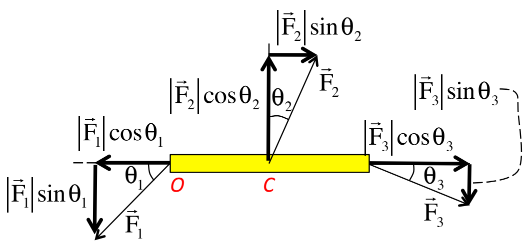
EXTRA: You are done with this problem, but it wouldn't hurt to note that there is another way (possibly easier?). You could have broken each of the forces into their component parts, then multiplied the components perpendicular to $\vec{r}$ by the magnitude of $\vec{r}$. Then:



$$\begin{aligned}
\vec{\Gamma}_{net,O} &= \vec{\Gamma}_1 + \vec{\Gamma}_2 + \left(-\vec{\Gamma}_3\right) \\
&= 0 + r_{2,\perp}\,|\vec{F}_2| - r_{3,\perp}\,|\vec{F}_3| \\
&= 0 + \left(|\vec{F}_2|\cos\theta_2\right)\left(\frac{L}{2}\right) - \left(|\vec{F}_3|\sin\theta_2\right)L \\
&= \left[(25.0\ \text{N})\cos 30^\circ\right](2.00\ \text{m}) - \left[(10.0\ \text{N})\sin 20^\circ\right](4.00\ \text{m}) \\
&= 30.0\ \text{N}\bullet\text{m}
\end{aligned}$$

2.)

Doing this for *Point C*:



$$\begin{aligned}
\vec{\Gamma}_{net,C} &= \vec{\Gamma}_1 + \cancelto{0}{\vec{\Gamma}_2} + \left(-\vec{\Gamma}_3\right) \\
&= \left(|\vec{F}_1|\sin\theta_1\right)\left(\frac{L}{2}\right) - \left(|\vec{F}_3|\sin\theta_3\right)\left(\frac{L}{2}\right) \\
&= \left[(30.0\ \text{N})\sin 45^\circ\right](2.00\ \text{m}) - \left[(10.0\ \text{N})\sin 20^\circ\right](2.00\ \text{m}) \\
&= 35.6\ \text{N}\bullet\text{m}
\end{aligned}$$

Either approach will work (I did the former because it was good review for doing torque calculations in polar notation, in general!).

6.)